A MORPHOLOGICAL STUDY OF THE FORM OF NATURE

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ABSTRACT

A process of recreating some forms of nature, including shells, horns, tusks, claws, and spiral plants, is herein described. The forms of nature based on spirals and ramification are generated not through the use of object data calculated by measurement, but through the use of an algorithmic structure based on the laws of nature. Although there are a myriad of forms to the shapes of nature, they are represented on the basis of one common principle, which can be expressed by means of the same mathematical expressions. The graphic software which is called "GROWTH" (Growth Rationale Object Work Theorem) has now been designed to create these forms automatically. GROWTH incorporates the rational, common principle of geometrical series which are found in the structute of biological objects.

CR categories: Animation/Dynamics/ Art

Key words and phrases: Biological law, GROWTH, spiral structure, similarity transformation, computer graphics, computer animation, Raster graphics

INTRODUCTION

We often find various forms which can be regarded as the form of design in the natural environment. A form which exhibits such regularity and order attracts our interest. The laws under which these forms operate are rules. These laws are rules which are physical factors regulating the process of formation. If these laws are assumed to be the principles whereby forms are created, we can anticipate that things having a similar form are the result of a common principle in operation.

This program can create spiral figures which are composed on a set of similar, but smaller, units, by entering the direction of growth on 3-D coordinate axes and accumulating these smaller figures in proportion to the growth rate, which is expanding in a series. In the exercise designed to simulate the growth of spiral plants, a more centripetal structure is generated by increasing the number, the angles, and the length of the branches in a series at any branching point. The use of GROWTH to generate natural veins and tendrils stemming from various plants can be described as follows. Shaded images rendered by this process are included, as are scientific illustrations and animated pictures explaining the morphological organization of living things.

In 1977, Eichi Izuhara advised me that the growth of shells occurs based upon simple and rational principles. In this paper, we will explain the principle of the spiral form mathematically and discuss the plants, snails, and horns which are herein used as models.

The form of nature will be proven not by measuring three-dimensional data of models but by explaining various rules of formation mathematically and by re-creating the models based on that explanation.

Shells' Growth Model

By observing the forms of fossils and various shells, one can see that shells share a common morphological structure and plan, or mechanism. Because they are governed by simple mathematical formula, the explanation of the formation technique by computer graphics became possible.

In this paper, emphasis is placed upon shells' morphology starting from the principle of the accumulation of shells rather than setting the living object's internal tissue's chemical parameter. Also, the research was done without considering genetic factors. There are various formation factors which contribute to the growth of shells; however, the purpose of this paper is to focus on the principle of morphological formation, which, in natural science, is considered to have a great influence, and to develop that morphological formation algorithm in computer graphics. Therefore, the research does not end only in a biological morphological study - it extends to the application of the research to the exploitation of new techniques in the formative arts.

The following is a description of shells' growth simulation. Thus, as a model for biological growth principle (formula), its application to computer graphics is explained.

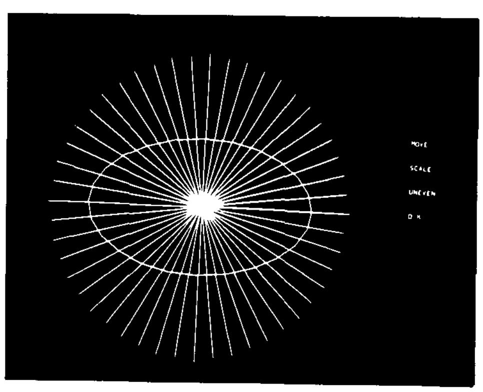


Figure 1 - Mouth of Shell

STRUCTURE ANALYSIS

Basically, shells have spiral structures supported by numerous small chambers which are arranged regularly.

Overall shell structure consists of small chambers.

Each chamber is similar

New chambers are accumulated at the position of previously accumulated old small chambers

Accumulation of the small chambers occurs locally. In other words, newly-formed small chambers are similar to those just previously formed. Therefore, small chambers are formed in an orderly and similar manner, and accumulation begins by geometic ratio to form a logarithmic spiral.

In order to determine these shells' overall morphological structure, only the small chambers' morphological structure has to be determined. Small chambers are polyhedrons which have similar top and bottom bases.

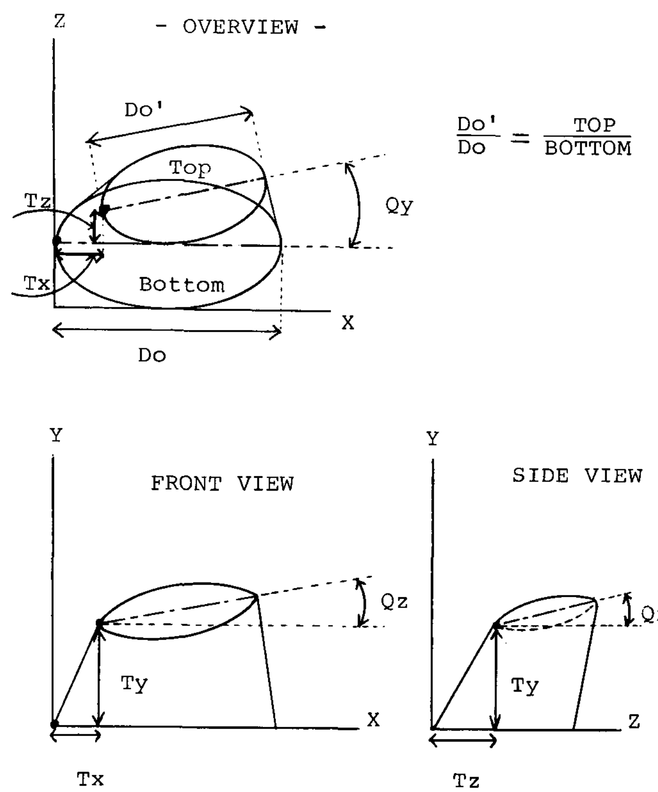


Figure 2 - Formation of Polyhedron

Algorithm of Three-Dimensional Small Chambers

I. Bottom base

Both top and bottom bases are polygons, which consist of N number apexes.

Formation of Polygons

1. Ellipse

The user has the option to input information by tablet and/or keyboard to vary the radius of the ellipse (see figure 1)

2. Non-elliptical configuration

The mouths of the shells which exist in nature have not only the shapes of ellipses but also have unique morphological structures.

Through the use of the above interactively, the concave/convex protuberance of the ellipse described above can be modified to construct the optimum shape of the shell's mouth.

II. Side View

Construct the mouth form which was constructed in a two-dimensional figure into a three-dimensional figure. At this point, the determined three-dimensional objects influence the overall shape of the shells. This, as I have described above, is composed of polyhedrons, which consist of two similar flat surfaces - bottom and top bases.

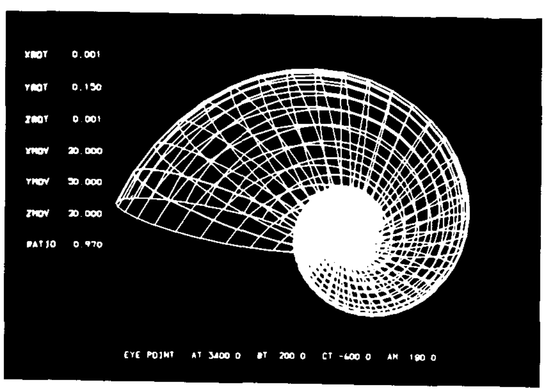


Figure 3 (above) Accumulated shell structure

Figure 4 (right, top) Horn; three views

Figure 5 (right, bottom) Horn, perspective view

Relationship Between Two Flat Surfaces

1) Identical Ratio

Make an identical figure of the enlarged or reduced scale of the bottom base, as was done at the top base.

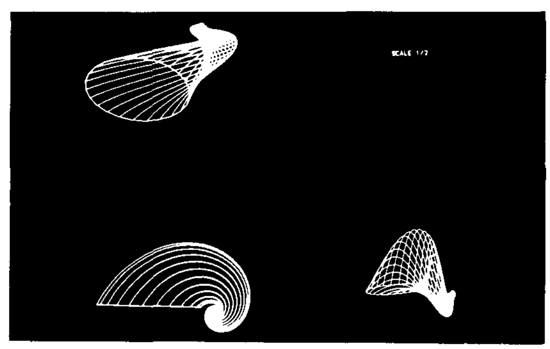
Supposing the long diameter of the ellipse to be $\underline{Do}$, and the scaled form of $\underline{Do}$ (either enlarged or reduced) to be $\underline{Do'}$. The ratio can thus be described as follows:

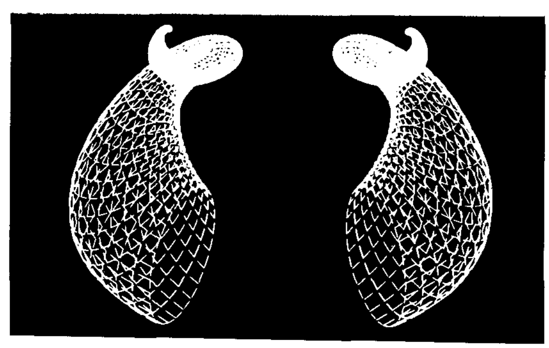
$$r = \frac{Do'}{Do}$$

This ratio (r) is the growth ratio of accumulation which takes place in each small chamber in the process of shell growth.

2) The Width

The Width is the distance between the top and bottom bases. Supposing the thickness to be the value of $\underline{H}$, as the figure of $\underline{H}$ is bigger, the shells with a rapid growth rate and a long and slender shape are constructed. The age of the shells is indicated by the number of small chambers. The relationship between the width and the position is a matter of the same level. That is, the width, $\underline{H}$, is the distance between the top and bottom bases, and the position is nothing but the transferring of axis $\underline{x}$ and $\underline{y}$, which reaches the top and bottom bases. Again, the top base is identical to the figure of the bottom base, in identical scale to it.





3) Inclination (Slope)

Inclination is indicated by the angle of the top and bottom bases of the shell. On a three-dimensional angular axis (x,y,z), it takes a direction either towards the right, left, hither or yon.

Actual shells have both right-twist and left-twist; however, they are not limited to either right or left twists.

To check the twist of shells in the process of data input, besides a three-dimensional picture consisting of a front-view picture, a side-view picture, and an overview, a picture done by perspective transformation is prepared (see figures 2, 3, 4). The reason for doing so is that there is a change in the direction of growth due to a change in the direction of the angle between the top and bottom bases. If the angle, $\underline{Q}$, between the top and bottom bases of the shells were small, the shells twirl a gentle, sloped curve spiral. On the other hand, if $\underline{Q}$ is big, shells twirl rapidly.

$$Q = (Qx, Qy, Qz)$$

4) Positional Relationship

If one observes shells in nature, he will find that the direction of the position is not necessarily vertical. Suppose the transfer distance against each direction of length, width, and depth to be $\underline{Tx}$, $\underline{Ty}$, and $\underline{Tz}$.

The relationship between the bottom base and the top base is indicated by the condition in which the distance of the height direction, i.e., the width relationship, $\underline{Tz}$, is fixed and moved in a certain direction. When the positional relationship of both bottom and top bases shifts from a vertical position to another position, each small chamber does not adjoin, as in the case of some shells, and it starts to form in the shape of horns of sheep or goats (see figure 5)

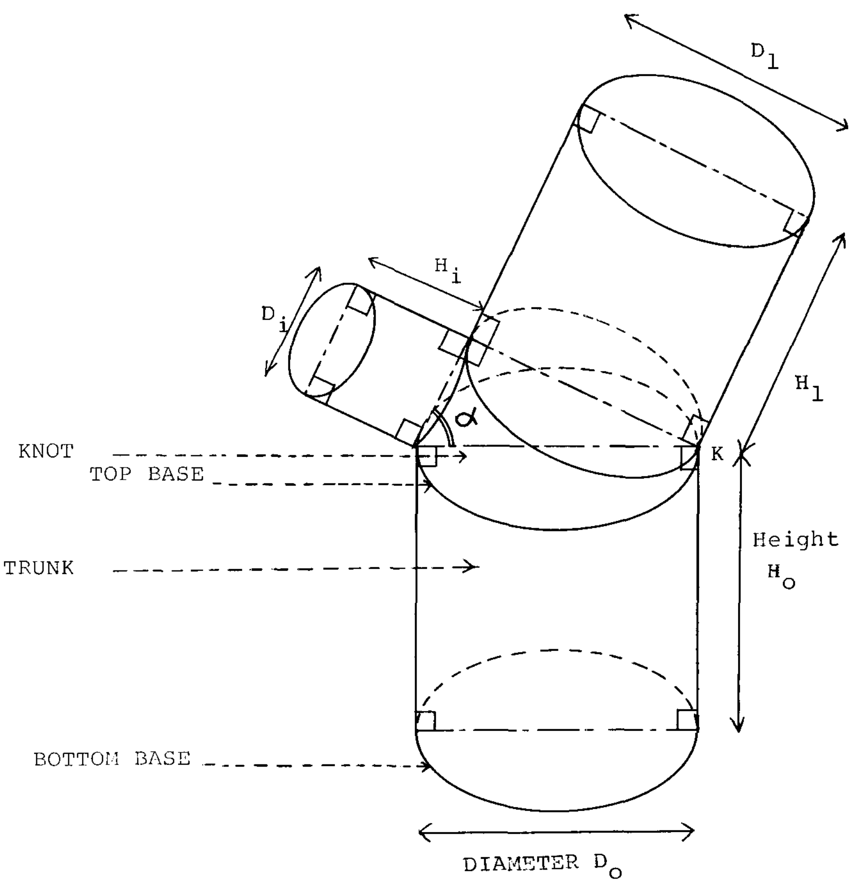


FIGURE 6 - Branching Structure

3. Growth Model by Branching

Making sure of the growth algorithm of a shell, we assume a knot between each chamber. For this purpose, a polyhedron with a top base and a bottom base which provides a previously-described chamber of the shell should be modified. This modification is accomplished such that the top base and the bottom base are always parallel, and if there is an angle between these bases it must be adjusted with a knot. By this modification no large difference is expected between this model and conventional growth model of a shell. Although we assume a knot, we can receive an essentially regular logarithmic spiral.

Several methods can be used in order to calculate the position of each chamber and knot. In this report, I have applied the theorem of a right-angled triangle by Pythagorus. As is well known, it is the basic theorem of algebra which states that the square of the slope on a right-angled triangle equals the sum of the squares of the other two sides. But by this simple theorem we are given a very important clue for solving the riddle of morphogenesis of natural formations such as shells, horns, nails, fangs, and plants.

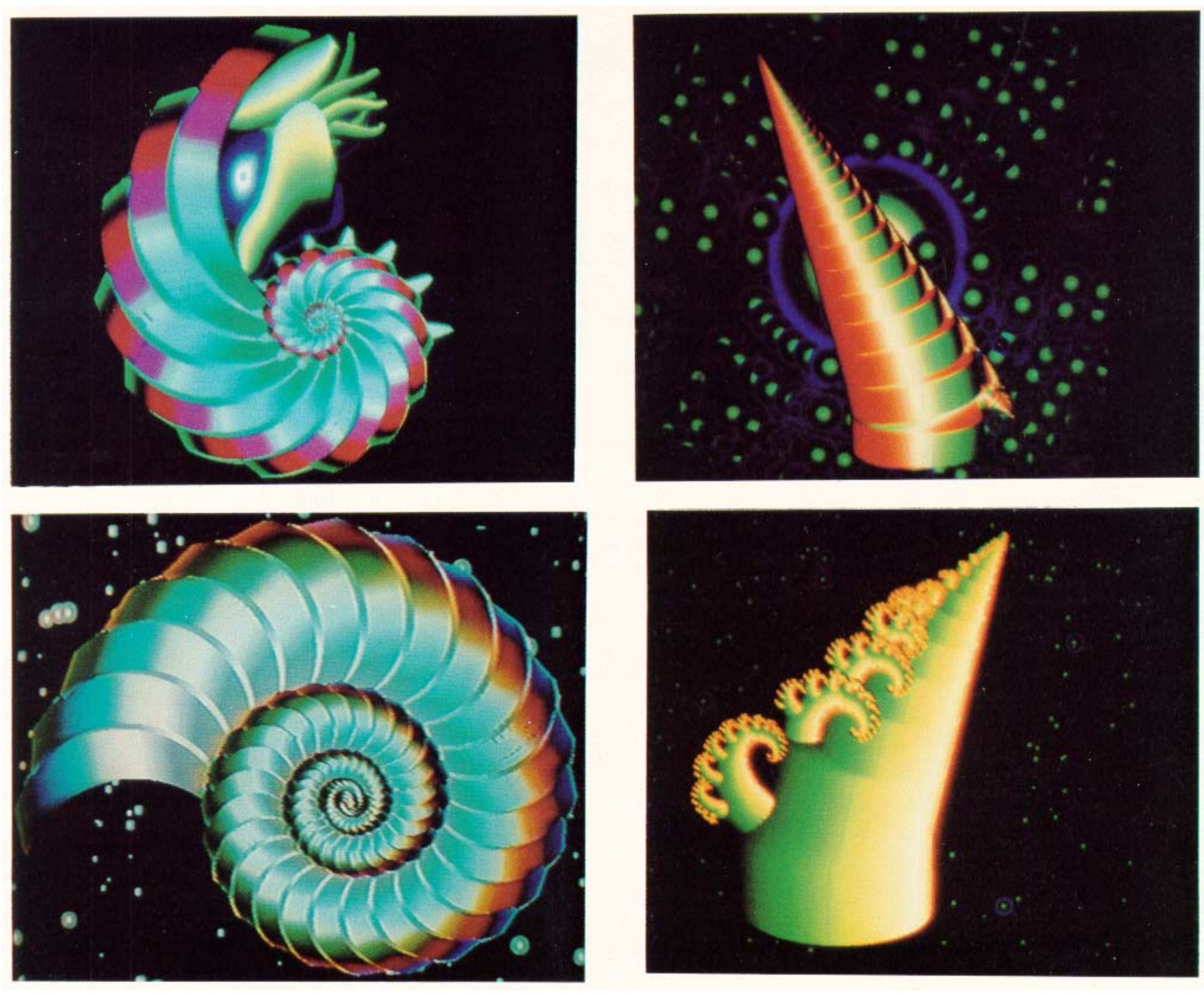


Figure 7 (left) Spiral Shells (Ammonite)
(right) Young Plant Growth

Rules of Generating Model

Simulation of the spiral growth of a plant is basically similar to the simulation of the growth of a horn, fang, nail, etc. The fundamental formation of the simulation contains a theory of the growth and some new, additional parameter which are factors in growth.

1) A trunk and branches deriving from it have diameters (D) and heights (H).
2) A top base and a bottom base of a trunk are parallel.
3) Two adjacent trunks are connected by a knot.
4) Between the top and bottom base of a knot is a certain angle (α)
5) The top and bottom base of a knot always contact each other at one point (K).
6) A branch always derives from a knot.
7) The diameter of a branch is always affected by the height of the knot from which it derives.
8) When the height of a branch or knot becomes smaller than one pixel, the branch stops growing.

i = the number of trunks
i = 0, 1, 2... n-1, n

$$\frac{Ho}{Do} = \frac{H1}{D1} = \frac{Hn}{Dn} = S \text{ (constant)}$$

(See figure 6)

3.2 Tendril Plants and Their Modification

A tendril plant can be generated by the application of the principle, which rolls a branch of a plant into a spiral. A tendril of a plant does not grow in a certain direction like the shell of a pearly nautilus, but a force which turns the growth to the other direction is generated at a certain rolling angle. We called this phenomenon the "self-adjustment law which a plant holds within itself" and made a model of a tendril plant by setting a limit to the angle of rolling and by operating the force which has the same strength and the opposite direction upon the growth of the plant. We could generate different types of tendril plant models according to the number of accumulation of bending. It is not necessary to input data like "twining around" or "sunward" of, for instance, a morning glory, to this simulation model. A plant itself generates new tendrils successively by self-multiplacation. If we shift not only the limitation of the angle for changing direction but also the value of the self-adjustment power, we will be able to get a plant which has a more complex curve. When we adjust the values at the number of bending to less than those of a tendril, a completed model more closely resembles a natural plant.

Extension of Generating Model

The following process is a morpho - genic model by which a plant successively repeats self-adjustment processes so as to generate a complex formation.

1) The basic structure shall follow the law of a growth of a tendril plant.
2) A rolling angle α shall gradually be increased or decreased at the rate of a certain value θ

$$\alpha(i) = \alpha(i) \pm \theta$$

3) A tendril derived from a knot shall receive a force β which adjusts the rolling angle of a trunk α

$$\alpha(i) = \alpha(i) \pm \theta \mp \beta$$

4) β shall have an effect upon α only when α is out of the provided limitations.

$$k_o < \alpha < k_1$$

5) After has been adjusted, will again come out of the provided limitations

$$k_o < \alpha < k_1$$

when accumulation reaches a certain number. Then the self-adjustment force will again operate upon to the opposite direction and try to recover a stable balance.

6) If we set the provided limitation extremely narrowly, we cannot see the growth of tendrils but will see the growth of crowded branches and leaves.

Thus we found it possible to generate a model which was very similar

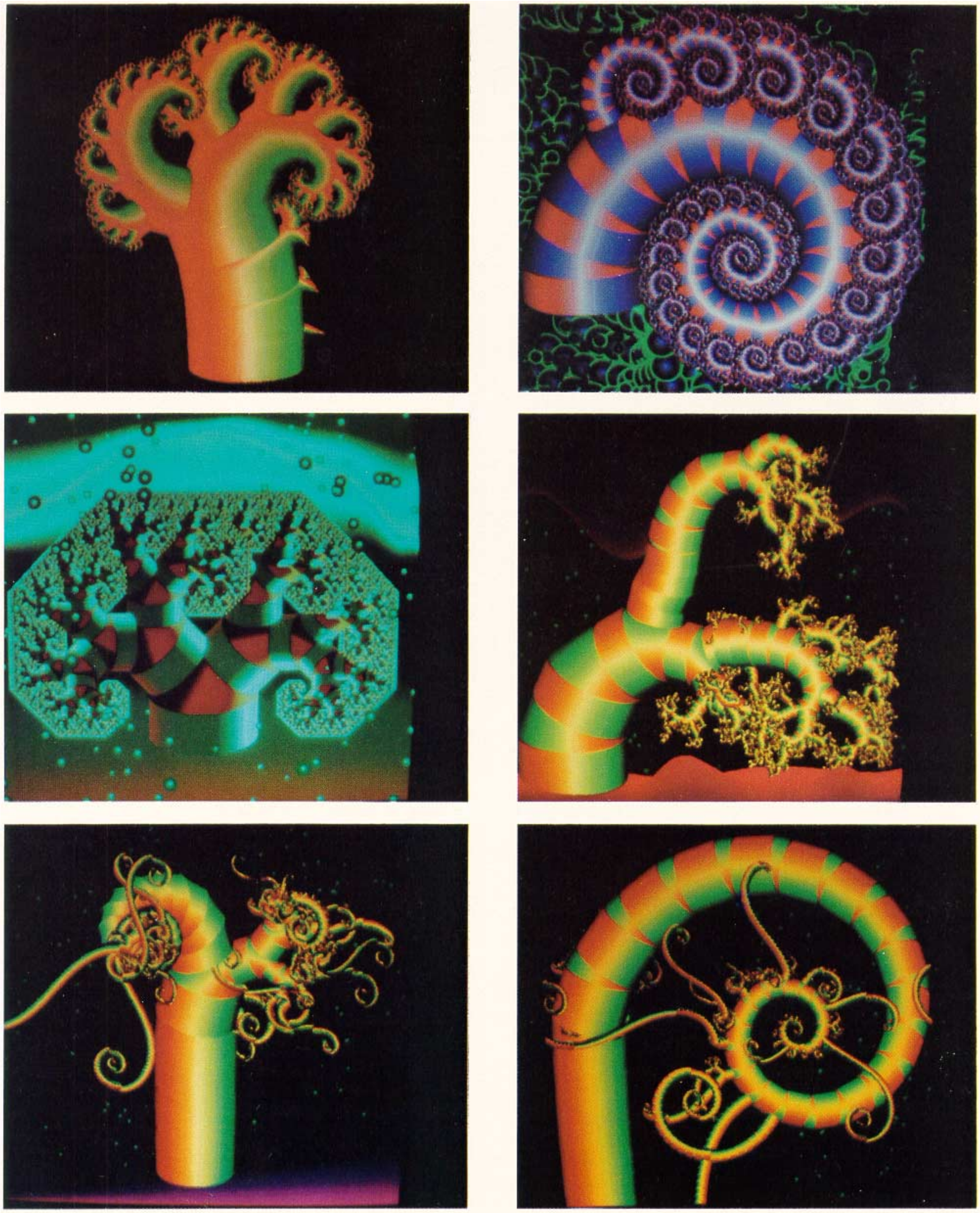


Figure 8 Spiral Growth - into Tendrils

to the existing plant by means of introducing a self-adjustment factor and other physical factors in addition to the basic principles of a spiral formation.

From the research reported this time, it is anticipated that, adapting the above result as a common theorem of the branching of plants, we will be able to construct various systematic models which express the evolution of plants from the past to the future, including plants existing at present.

CONCLUSION

Since this program possesses an algorithic structure based on the similarity growth that takes place in the natural world, it can create shells and horns of intended shape.

Shapes thus created are formed by curved surfaces obtained not through complex measurements but by the principles of regular logarithmic spiral.

By expanding these principles, we have succeeded in freely generating such complex shapes as corals and tendril plants, of which dimension is hard to measure by methods currently available.

The process to generate these shapes which might consist of hundreds of thousands, or even millions of faces, is an application of biological and mathematical principles by branching.

The equipment used in this research are as follows:

E & S Picture System II and Melcom 700 for line drawings

AED 512 and Micro - 11/23 for shaded images

The language was FORTRAN

A background picture and a picture of a shell or a group of spiral plants are made separately

6. Acknowledgements

I started this research in 1977, with the advice of Professor Eiichi Izuhara at the Industrial Product Research Institute, and the first step was a growth model of a shell by line drawing. At that time, Mr. Tomohiro Ohira, also at the Industrial Research Institute, gave me suggestions concerning the design of three-dimensional curved surfaces. I would like to express my sincere thanks to both of them.

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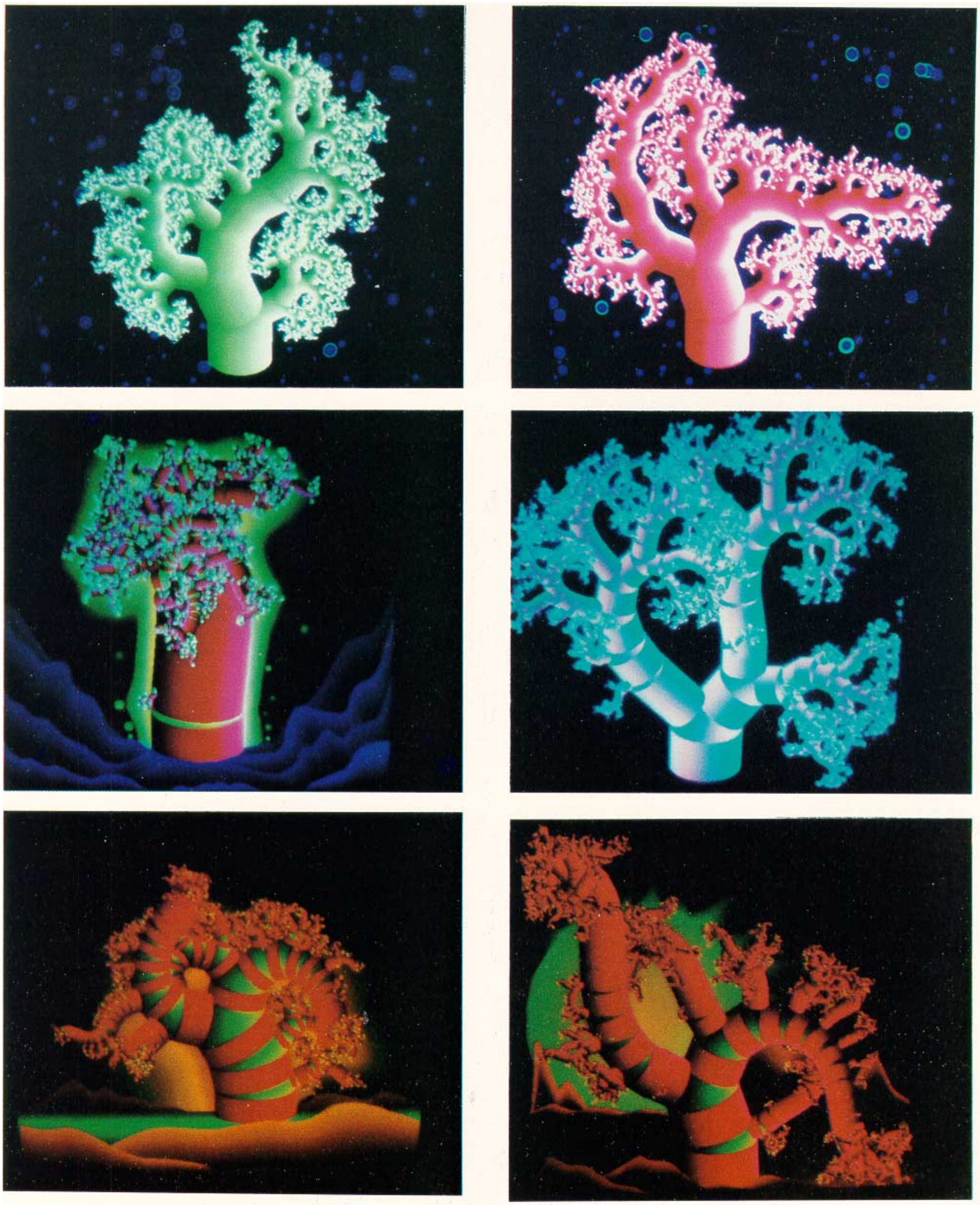

Figure 9 Coral Growth Patterns

Figure 10 Tentacles Growth Patterns